

A NEW EMPIRICAL STATISTICAL METHODOLOGY FOR DETECTION OF PARALLELISMS
AND DATING OF DUPLICATES

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In this article we present in greater detail the results originally published in Dokl. Akad. Nauk SSSR [1, 2], which were obtained by the application of one of the procedures developed by the author and reported at the Third International Vil'nyus Conference on Probability Theory and Mathematical Statistics [3, 4]. These seven procedures are applicable to historical texts and may be used to date ancient events and to detect dependent texts, duplicates, or parallelisms. In particular, one of the procedures enabled us to process the rich material representing the sequences of lengths of rule in various ancient dynasties.

In his seminal study [5], Morozov pointed to three pairs of dynasties for which the length of rule was represented by visually similar graphs. This could imply that in reality these dynasties were reflections of the same real dynasty "replicated" as a result of erroneous dating of various lists (texts) describing the same events. At the same time, my analysis in [6] revealed about 10 allegedly independent pairs of dynasties with such and even more strikingly similar graphs. It therefore became necessary to develop a formal methodology capable of distinguishing quantitatively (and not merely by visual likeness) between dependent and independent dynasties.

Let us state this problem in rigorous terms. Consider n successive real rulers in the history of some region, where n is a fixed integer, e.g., $n = 15$. We will refer to this sequence as the real dynasty. Each chronicler describing this dynasty A uses his own method to compute the length of rule a_i of ruler i and thus generates a sequence of numbers $\alpha = (\alpha_1, \dots, \alpha_n)$, while we will call the numerical dynasty. It is conveniently represented by the vector α in the Euclidean space R^n . Another chronicler describing the same real dynasty in general comes up with a different numerical dynasty $b = (b_1, \dots, b_n)$. The difference is essentially attributable to errors and objective difficulties which prevent exact determination of the actual lengths of rule. The main types of errors actually made by chroniclers were identified and described in [1, 6]. Thus, each real dynasty in general may be "replicated" by various chroniclers into several different numerical dynasties. Each real dynasty A may be represented by some set $V(A)$ of points (vectors) in R^n . The "fuzziness" of this set shows how large were the errors of the various chroniclers. Large errors result in a set $V(A)$ with widely scattered points. If the errors are small, $V(A)$ has a small "diameter."

How large were the errors that chroniclers made in the description of real dynasties? The author formulated and tested in [1] the principle of small distortions, which provides an answer to this question. Let us briefly review this principle. Two numerical dynasties are said to be dependent if they represent the same real historical dynasty, i.e., constitute two descriptive versions of the same dynasty. Conversely, two numerical dynasties are called independent if the two real dynasties that they describe are essentially different. Real dynasties are regarded as being essentially different if the number of rulers occurring in both dynasties simultaneously does not exceed $n/2$, i.e., half the number of rulers in the particular dynasty. Note that two arbitrarily chosen dynasties may intersect, i.e., some rulers may simultaneously occur in both dynasties. Thus, alongside dependent and independent pairs of numerical dynasties, we also have "intermediate" pairs of dynasties in which the number of common rulers is greater than $n/2$. If the total number of dynasties is large, the number of intermediate pairs of dynasties is relatively small and we can mainly focus on dependent and independent pairs of dynasties. The principle of small distortions is stated as follows: If the difference between two numerical dynasties is "small," then they are dependent, i.e., represent the same real dynasty; if, on the other hand, two numerical dynasties are independent (i.e., represent essentially different real dynasties), then they "significantly" differ from one another. This principle (conjecture) claims that real chroniclers on the whole make

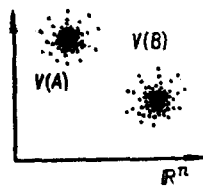


Fig. 1

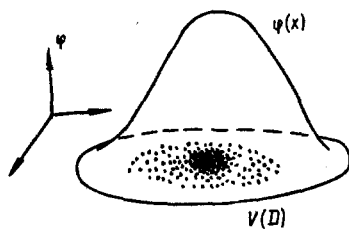


Fig. 2

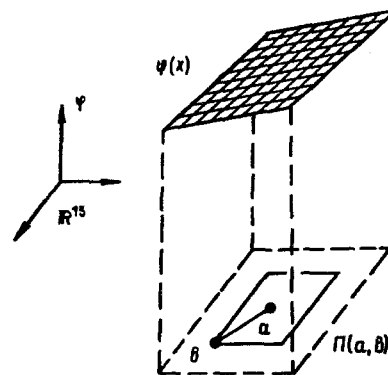


Fig. 3

small errors, i.e., they introduce small distortions into real numerical data in their chronicles.

This principle can be visually represented in terms of the sets $V(A)$ and $V(B)$ in R^n . For each real dynasty A, the set of corresponding numerical dynasties constitutes a "spherical cluster" $V(A)$ (Fig. 1). If our principle holds, the spherical clusters $V(A)$ and $V(B)$ corresponding to different real dynasties A and B do not overlap, i.e., they are sufficiently distant from one another. Our principle naturally requires empirical verification. Is there a natural numerical coefficient (measure) $\lambda(a, b)$ which can be evaluated for each pair of numerical dynasties a and b so that it is "small" for dependent dynasties and "large" for independent dynasties? In other words, this coefficient should clearly discriminate between dependent and independent dynasties. Such a coefficient (a measure on the set of dynasty pairs) was proposed by the author in [1]. The principle of small distortions was verified by using specific historical material. The existence of such a natural measure $\lambda(a, b)$ is not *a priori* self-evident: Its existence is nontrivial and it discloses an interesting law which characterizes all the numerical data of ancient chronicles surviving to this day.

Let us briefly recall the definition of $\lambda(a, b)$. By processing some 300 historical chronicles, records, primary sources, tables, etc., the author compiled a list of all the known numerical dynasties from 4000 B.C. to 1800 A.D. in the history of Europe, the Mediterranean, Near East, and Egypt. Then a complete list of all 15-ruler dynasties was compiled, i.e., tables of all groups of 15 successive rulers [1, 6]. Each dynasty was then represented by a point (a vector) in R^{15} , and we thus obtained some set D . The most typical chronicling errors were simulated, such as a) transposition of two successive rulers, b) substitution of one ruler for two successive rulers, reporting his rule to be equal to the sum of the lengths of rule of the two actual rulers, c) errors in the computation of the length of rule: the longer the rule, the greater the computational error. Each dynasty from the set D was subjected to all the perturbations a) and b). Each point in D was thus replicated into several points, and the set D on the whole increased. We denote the resulting set of $15 \cdot 10^{11}$ points by $V(D)$. Then the set $V(D)$ was used to derive a probability density function φ . To this end, the space R^{15} was divided into sufficiently small standard cubes so that none of the points in $V(D)$ was located on the boundary of any of the cubes. If x is an interior point of a cube, we set $\varphi(x) = [\text{the number of points from } V(D) \text{ contained in the cube}] / [\text{total number of points in } V(D)]$. If x is on the boundary of a cube, we set $\varphi(x) = 0$. This function φ has its maximum in the region where particularly numerous dynasties from $V(D)$ are clustered and falls off to zero where no points from $V(D)$ are observed (Fig. 2).

The measure $\lambda(a, b)$ now can be defined as follows. Let $a = (a_1, \dots, a_{15})$ and $b = (b_1, \dots, b_{15})$ be two numerical dynasties from $V(D)$. Construct the parallelepiped $\tilde{\Pi}(a, b)$ centered at the point a whose diagonal is the vector $b - a$ (Fig. 3). Now allow for chronicling errors of type c), i.e., errors made by real chroniclers when computing the length of rule of different rulers. Expand the parallelepiped $\tilde{\Pi}(a, b)$ by enclosing it in a larger parallelepiped $\Pi(a, b)$, whose orthogonal projections on the coordinate axes are segments with end points $a_i \pm |a_i - b_i| + f(a_i)$, where f is the chronicling error function — a nondecreasing positive function of a_i [1, 6, 13]. For $0 \leq a_i < 20$, we have $f(a_i) = 2$, for $20 \leq a_i < 30$, we have $f(a_i) = 3$; for $30 \leq a_i < 100$, we have $f(a_i) = 5 \cdot [a_i/10] - 5$. Now $\lambda(a, b)$ is defined as the number $\int_{\Pi(a,b)} \varphi(x) dx$. In

Fig. 4 the number $\lambda(a, b)$ is depicted by the volume of the prism whose base is the parallelepiped $\Pi(a, b)$ and which is bounded from above by the graph of the function φ .

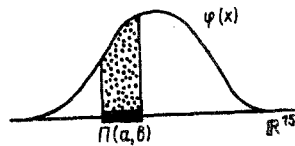


Fig. 4

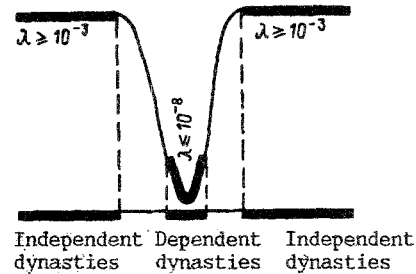


Fig. 5

The number $\lambda(\alpha, b)$ may be interpreted as the probability that the distance from the point α of the random vector ξ distributed in the space \mathbb{R}^{15} with the density function Ψ does not exceed the distance between α and b . For further details, see [1, 6, 13]. An extensive numerical experiment subsequently showed that for known dependent numerical dynasties α and b , the coefficient $\lambda(\alpha, b)$ varies from 10^{-12} to 10^{-8} . The experiment was conducted using known dependent dynasties from historical chronicles between 4000 B.C. and 19th century A.D. Conversely, for known independent numerical dynasties between 13th century A.D. and 19th century A.D., the number $\lambda(\alpha, b)$ was found to be no smaller than 10^{-3} (Fig. 5).

The principle of small distortions was thus fully corroborated using reliably dated historical material for the period between 13th and 19th century A.D. In particular, the application of our dating methodology to the period between 13th and 19th century A.D. failed to detect any discrepancies in the traditional dates of historical dynasties.

Our algorithm is applied as follows to compare two real dynastic streams (i.e., two sequences of all rulers in a given region). From each stream of rulers we select a dynastic strand, i.e., a subsequence of rulers who cover the overall time interval corresponding to the given dynasty. Since real dynastic streams often contain many corulers, several different strands (numerical dynasties) may be selected from any given stream.

Moreover, for each ruler we consider all the possible beginning and ending dates of his rule. Again recall that these dates are sometimes recorded differently by different chroniclers, and therefore all the possible versions should be taken into consideration when selecting the strands. This factor also increases the number of possible strands extracted for a given dynasty.

In a comparison of two dynasties, it may turn out that only two strands are dependent from among all the possible pairs of strands, while the remaining strands are independent. It is significant that when comparing the dynasties (or strands) a and b , the lengths of rule are treated as rough approximations, since the error function $f(a_i)$ is built into the algorithm. In other words, if the length of rule a_i is less than 20 years, the difference $a_i - b_i$ is considered only to within ± 2 years; if a_i varies between 20 and 30 years, then the difference is taken to within ± 3 years. If a_i exceeds 30 years, the admissible error may be ± 10 years, increasing linearly thereafter with the increase of $[a_i/10]$ (the whole part). Therefore, the algorithm only requires roughly approximate lengths of rule, and not their exact values, which in most cases are unknown.

The character of the graph plotting the length of rule, i.e., the shape of the polygonal curve representing this graph, is also of importance. The algorithm itself and the results obtained by this procedure are seen to be highly stable with respect to errors in the length of rule between the above limits.

The application of this methodology to historical material traditionally dated before 13th century A.D. unexpectedly detected pairs of dynasties (strands) a and b which are regarded as independent (in all senses) and yet their closeness coefficient $\lambda(\alpha, b)$ is of the same order of magnitude as for known dependent dynasties, i.e., not greater than 10^{-8} . In this article, Tables 1-13 list the rulers (according to traditional dating) which are included in the most interesting of these special pairs of dynasties that we discovered. For all these pairs of dynasties, $\lambda(\alpha, b) < 10^{-8}$, i.e., they are dependent, duplicates or parallels of one another.

In graphical analysis, we compare the graphs of the length of rule in the given sequence of rulers and also examine the superposition of two dynasties on the time axis by rigidly shifting one dynasty until it overlaps the other. As a result of this rigid shift, the

TABLE 1 (see Fig. 6 below)

Carolingians, the empire of Charlemagne, 7th-9th cent. A.D. Shift by 360 years. For variant rules see [14,15]

- 1) Pepin of Herstal 681-714 (33)
 - 2) Charles Martel 721-724 (20)
 - 3) Pepin the Short 754-768 (14)
 - 4) Charlemagne 768-814 (46)
 - 5) Carloman 768-771 or 772 (3 or 4).
- Charlemagne's famous "gift" (774): gives the lands of Italy to the Pope
- 6) Louis I the Pious (814-833) (abdication) (19)
 - 7) Lothair the Western 840-855 (15)
 - 8) Charles the Bald 840-875 (35)
 - 9) Louis the German 843-875 (32)
 - 10) Louis II the Western 855-875 (20)
 - 11) Charles the Fat 880-888 (8). The breakup of the Carolingian empire (the West). A shift of some 360 years. War

A strand from the Third Roman Empire, 3rd-4th cent. A.D. (mainly Eastern). For variant rules see [14,16]

- 1) Constantius II 324-361 (37)
 - 2) Theodosius I 379-395 (16)
 - 3) Arcadius 395-408 (13)
 - 4) Theodosius II 408-450 (42)
 - 5) Constantine III 407-411 (4).
- The famous "gift" of Constantine I the Great (4th century A.D.): gives Rome to the Pope
- 6) Leo I 457-474 (17). The Carolingians on the left "recreate antiquity"
 - 7) Zeno 474-491 (17)
 - 8) Theodoric 493-526 (33)
 - 9) Anastasius 491-518 (27)
 - 10) Odoacer 476-493 (17)
 - 11) Justin I 518-527 (9). Breakup of the official Third Roman Empire in the West. Gothic war of 4th cent. A.D.

A mean shift by ends of rule is 359.6 years, which coincides with a rigid shift of 360 years for overlapping of left and right columns.

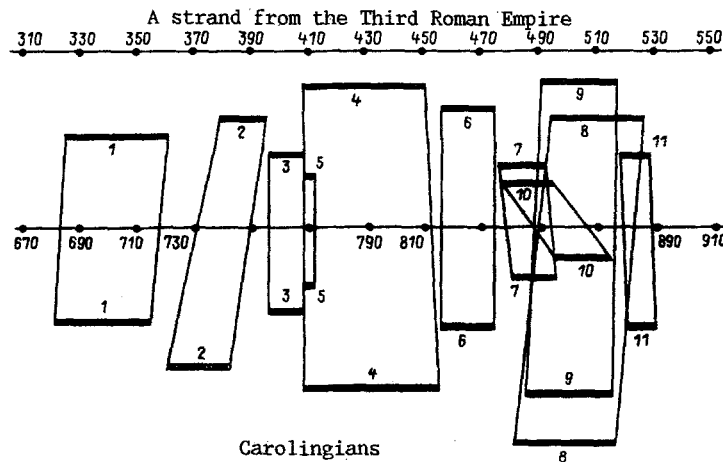


Fig. 6

relative position (in time) of the individual rulers is preserved. For visual presentation, vertical segments are drawn joining the beginnings (and also the ends) of rule of the superimposed rulers. Here we only give some basic examples. We also computed the mean shift, equating the ends of rule of the superimposed rulers.

The parallelism of all these dynasties (see Tables 1-13 and Figs. 6-14) is perfectly consistent with the author's discovery that the global chronological chart GCC (i.e., the modern "textbook" of ancient and medieval history; see definitions and description in [1-4, 6-13]) decomposes into four identical chronicle strata. Table 14 presents a more detailed description of the GCC than that in [2] (see [2, Fig. 3]). The line E (the left-hand column) represents the ancient and medieval history of Europe, the Mediterranean, and Near East using traditional dating. Line B represents the biblical chronology and history according to the books of the Old and the New Testaments. This history is presented with an upward shift, highlighting its superposition on the events in European history as discovered by the author. The letters K, T, II, P, C, H in Table 14 and in Fig. 3 in [2] represent various historical epochs (periods). For brevity, we relabeled with the letter T the epochs that were marked in Fig. 3 in [2] by black triangles with the letters MT. The line C₀ in Table 14 is the original, i.e., the chronicle which in all probability describes the actual history of the regions and the true chronology (the first line from the bottom in Fig. 3 in [2]). The line C₁ (3rd line from the bottom in Fig. 3 in [2]) presents a distorted original C₀ containing several duplicates. The lines C₂, C₃, C₄ in Table 14 (respectively, the 4th, 5th, and 6th lines from the bottom in Fig. 3 in [2]) represent the line C₁ shifted downward in time by 333 years, 1053 years, and 1778 years, respectively. Thus, the events in Table 14 labeled by the same numbers and the same letters, i.e., located in the same horizontal positions in the

TABLE 2 (see Fig. 7 below)

Roman German Empire 10th-13th cent. A.D.
Variant rules see [14,15,17]

- 1) Otto III the Red (-Chlorus) 983-1002 (19). A duplicate of Julius Caesar 1st cent. B.C.
- 2) Henry II 1002-1024 (22)
- 3) Conrad II 1024-1039 (15)
- 4) Henry III 1028-1056 (28)
- 5) Henry IV 1053-1106 (53). During his rule, Hildebrand from 1049(beginning of activity in Rome) to 1085 (death) (36 years) = the original of Basil the Great. In 1053, the beginning of Hildebrand's famous reform; then his contest with Henry IV (Canossa)
- 6) Henry V 1098-1125 (27)
- 7) Lothair 1125-1138 (13)
- 8) Conrad III 1138-1152 (14)
- 9) Frederick I 1152-1190 (38)
- 10) Henry VI 1169-1197 (28)
- 11) Anarchy and Philip of Ghibelline 1198-1208 (10). Contemporaries: Suburus, Petrus, Rainerius
- 12) Otto IV 1207-1217 (17 or 16) or 1197-1218 (21). Capture of Rome and coronation. Otto IV is a German
- 13) Frederick II as a Roman emperor 1220 (final coronation)-1250 (30). The execution of Vinea (= Boethius?)
- 14) Or: Frederick II 1196-1250 (54) and co-ruler Otto IV until 1218. Frederick's death is the beginning of the war in Italy in 13th cent. A.D.
- 15) Conrad IV 1237-1254 (17). Opponent Charles of Anjou
- 16) Manfred 1254-1266 (12)
- 17) Conradin (very young) 1266-1268(2). Died in Naples. Lost the battle against Charles of Anjou near Troy and Naples. End of the Empire 10th-13th cent. A.D.. Destruction and fall of the Hohenstaufens.

Third Roman Empire 4th-6th cent. A.D. Variant rules see [14,16,17]

- 1) Constantius I Chlorus 293-306 (13). A duplicate of Julius Caesar (a shift of 340 years)
- 2) Diocletian 284-305,304 (21)
- 3) Licinius 308-324 (16)
- 4) Constantine I 306-337 (31)
- 5) Basil the Great (?), 333-378 (45) The shift from Henry to Basil is 728 years = 1106-378. The shift from the "birth" of Hildebrand to Basil 1053-333 = 720 years. The famous reform of Basil (basileus?). Contest of Basil with Valens (Herod?)
- 6) Honorius 395-423 (28)
- 7) Theodosius I 379-395 (16)
- 8) Arcadius 395-408 (13)
- 9) Theodosius II 408-450 (42)
- 10) Valentinian III 425-455 (30)
- 11) Anarchy and Ricimer 456-472 (16). Contemporaries: Severus, Petronius, Recimir. See two close names on the left.
- 12) Anarchy and Odoacer 476-493 (17). Capture of Rome and coronation. Odoacer is the leader of German gerulians.
- 13) Theodoric 497-526 (29). A variant from [17]. Note closeness of names Theodoric and Frederick
- 14) Or: Theodoric - Odoacer (co-ruler) 476-526 (50). Theodoric's death is the beginning of the war in Italy in 4th cent. A.D. (the Gothic).
- 15) The Gothic dynasty 526-541 (15). Opponents: Belisaruis, Narses
- 16) Totila 541-552 (11)
- 17) Teias (very young) 552-553 (1 or 2). Died near Naples. Lost the battle against Narses near Troy and Naples (= Trojan War). End of Third Roman Empire in Italy. Destruction and fall of the Goths.

Mean shift by ends of rule is 723 years, which is close to the rigid shift of 720 years for overlapping the left and the right columns.

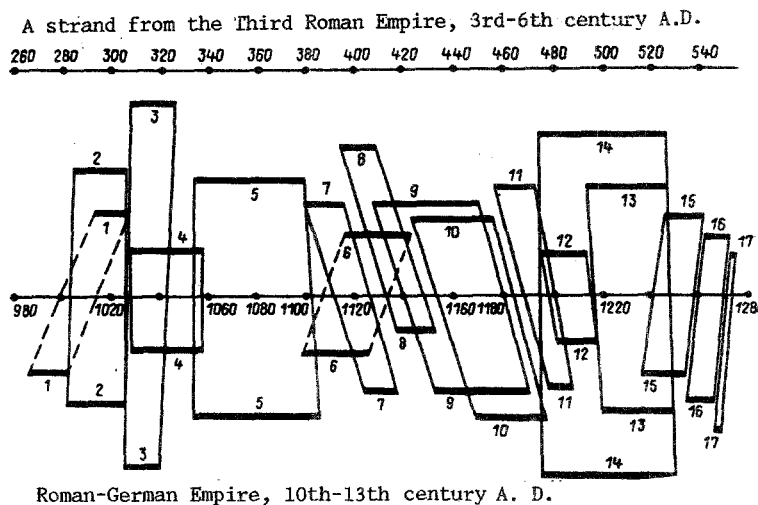


Fig. 7

TABLE 3 (see Fig. 8 below)

Roman German Empire 10th-13th century A.D. The beginning of the Saxon dynasty in 911. Variant rules in both columns, see [14,15,18]

- 1) Conrad I 911-918 (7)
- 2) Henry I 919-936 (17)
- 3) Otto I the Great 936-973 (37).
Superimposed on Alberic II = Albrecht I
- 4) Otto II from the death of Otto I in 973 to his death in 983 and Otto III 983-1002 (total of 29 years)
- 5) Henry II 1002-1024 (22)
- 6) Conrad II from coronation in Rome in 1027 to his death in 1039 (2)
- 7) Henry III the Black 1028-1056 (28). The great split of the Church under Hildebrand
- 8) Henry IV 1053-1106 (53)
- 9) Henry V 1098-1125 or Henry V from the coronation in Rome in 1111 to his death in 1125 + Lothair II 1125-1137 (27 or 26)
- 10) The eruption of Vesuvius in 1138-1139. War in Italy with the participation of Germany 1143-1155. The revolt of Arnold of Brescia.
- 11) Frederick I Barbarossa 1152-1190. Famous emperor. Rome captured by Frederick in 1154. Difference in dates 373 years (shift). Pope Adrian IV. Establishment of Franciscan and Dominican orders (1223 and 1220)
- 12) Henry VI 1191 (coronation in Rome) - 1197 (6)
- 13) Philip 1198-1208 (10)
- 14) Frederick II 1211-1250 (39). Three coronations 1196,1211, 1220
- 15) Wilhelm 1250-1256 (6)
- 16) Conrad IV 1237-1254 (17)
- 17) End of the Empire 1250-1254
- 18) War in Italy 1250-1268. Beginning of the 17-year Interregnum in Germany (1256)

Habsburg Empire 13th-17th century A.D. The beginning of the Austrian line in 1273. A rigid shift of some 362 years ensures overlap with the column on the left.

- 1) Adolf of Nassau 1291-1298 (7)
- 2) Rudolf of Habsburg 1273-1291 (18)
- 3) Henry VII 1309-1314 and Louis V 1314-1347 (total of 38)
- 4) Charles IV 1347-1378 (31). To No. 10: in the Habsburg Empire there were only three eruptions of Vesuvius, in 1306, 1500, and 1631.
- 5) Wenzel 1378-1400 (22)
- 6) Rupert of the Palatinate 1400-1410 (10)
- 7) Sigismund 1410-1438 (28). The great split of the Church in 1378-1417, see the books of Ezra, Nehemiah, and Esther in the Bible.
- 8) Frederick III 1440-1493 (53)
- 9) Maximilian I Pius 1493-1519 (26). In his time, "Almagest" of Ptolemy from the time of Antoninus Pius (138-161). Overlap by a shift of 1000 + 300.
- 10) The eruption of Vesuvius in 1500. The beginning of the Italian wars of Germany 1494-1527. Brescian revolt in 1512 (see left column).
- 11) Charles V 1519-1556 (37). Famous emperor. In his time, Frederick the Wise and war with Barbarossa. Rome captured by Charles V in 1527. Pope Adrian VI. Establishment and official confirmation (1540) of the Jesuit order.
- 12) Ferdinand 1556-1564 (6)
- 13) Maximilian II 1564-1576 (12)
- 14) Rudolf II 1576-1612 (36)
- 15) Matthias 1612-1619 (7) (Matthew)
- 16) Ferdinand II 1619-1637 (18)
- 17) End of the Empire 1618-1619
- 18) Breakout of the Thirty-Year War in Germany in 1618.

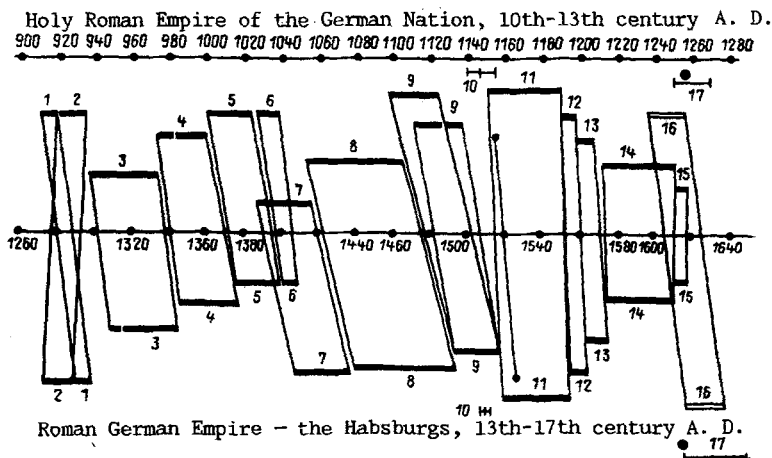


Fig. 8

table, are duplicates, i.e., events which are identified as (completely or partially) coinciding by the new dating methods. Events labeled simultaneously by several letters in the first and the second columns of the table are the sum (superposition) of the events entered in the same horizontal positions in the other columns of the table under the same label. For instance, the events No. 16, P/C in the first column of Table 14 are obtained by super-

TABLE 4 (see Fig. 9 below)

Holy Roman Empire of the German Nation in Italy 10th-13th century A.D. Duration 292 years (962 or 964 to 1254). Variant rules see [14, 16-19] (right and left)
 The beginning of the Empire; the three great emperors of 10th century:
 Otto I the Great (anarchy and war)
 Otto II the Wild
 Otto III the Red (Chlorus)
 A) Otto I as a German king: 936-973 (37). Octavian, son of Alberic (duplicate of Julius Caesar), accedes to power at the age of 16 (young).
 B) Otto II 960 (German coronation) 983 (23)
 C) The emperors are German kaisers. There are virtually no gold imperial coins of 10th-13th century. A.D. Possibly attributable to the right column. Empire officially known as Holy

- 1) Henry II the Holy + Conrad the Salian 1002-1039 (37)
- 2) Conrad II the Salian 1024-1039 (15). In his time Pope Hildebrand in Rome 1053-1073-1085. The betrayal of Cencius, "Papal passion" [17]
- 3) Henry III the Black 1028-1056 (28)
- 4) Henry IV 1053-1106 (53). The names on the right are close: all contain a common part Tiberius Claudius Nero and Germanicus
- 5) Henry V the Black 1098-1125, German king (27) ? or No. 6
- 6) Henry V the Black 1111-1125, Roman emperor (14)
- 7) Lothair 1125-1137 (12)
- 8) Vesuvius erupts 1138-1139 (duplicate of the 1500 eruption?)
- 9) Conrad III 1138-1152 (14)
- 10) Frederick I Barbarossa 1152-1190 (38). Chronicles confuse with Frederick II
- 11) Henry VI 1169-1197 (28)
- 12) Philip of Ghibelline 1198-1208 (10)
- 13) Otto IV 1198-1218 (20). In his time the famous column for Marcus Aurelius was erected (see [17], vol. 4, p. 568, note 74)
- 14) Frederick II 1211-1250 (39). His title is Gattin (the Goth?)
- 15) Conrad IV 1237-1254 (17)
- 16) Interregnum 1256-1273 (17). End of the Roman Empire of 10th-13th century A.D. War in Italy in mid-13th cent. Duplicate of Gothic (Trojan) war.

Second Roman Empire from 1st cent. B.C. to 3rd cent. A.D. in Italy. Duration 299 years, from 82 B.C. to 217 A.D. Overlap observed with upwrd rigid shift of 1053 years
 The beginning of the Empire; the three great emperors of 1st cent. B.C.:
 Pompey the Great (anarchy and war)
 Sulla Lucius (transposed with No. 1)
 Julius Caesar (- Chlorus in Third Empire)
 A) Octavian Augustus 23 (or 27) B.C. to 14 A.D. (37). Octavian, adopted son of Julius Caesar, accedes to power at the age of 19 (young).
 B) Tiberius 14-37 (23). Reason for the shift: 11th century A.D. - 10th century of Jesus Christ (1st century).
 C) The emperors are caesars, often bearing the name Germanicus. Many gold coins are known from ancient Rome before the Second Empire. The emperors are often called Augustus (Holy).

- 1) Octavian Augustus (the Holy) 23 B.C.-14 A.D. (37)
- 2) Germanicus 6-19 (13). In his time, Jesus Christ 0-33, a duplicate of Hildebrand under a shift of 1053 years. The betrayal by Judas, "Christ's passion."
- 3) Tiberius + Caligula 14-41 (27)
- 4) Tiberius + Caligula + Claudius + Nero 14-68 (54)? A doubtful join.
- 5) Claudius + Nero 41-68 (27)? Full name includes "Black."
- 6) Nero 54-68 (14). No joins in this variant
- 7) Two Titus Vespasians 69-81 (12)
- 8) Vesuvius erupts, destruction of Pompeii and Herculaneium in 79 A.D.
- 9) Domitian 81-96 (15)
- 10) Trajan + Hadrian 98-138 (40). Both named Trajan (join by name)
- 11) Antoninus Pius 138-161 (23)
- 12) Lucius Verus 161-169 (8)
- 13) Marcus Aurelius 161-180 (19). The equestrian statue of Marcus Aurelius is famous ancient Roman relic. Confusion in medieval chronicles [17].
- 14) Commodus + Caracalla 180-217 (37), duplicate of Theorodic the Goth in 6th cent.
- 15) Septimius Severus 193-211 (18)
- 16) Anarchy, Julia Maesa and her favorites 217-235 (18). End of Second Roman Empire. War with the Goths.

Mean shift by ends of rule 1039 years, which is close to the rigid shift of 1053 years overlapping left and right columns.

position (summing) of the events No. 16, C from the line C₂ and No. 16, P from the line C₁. Table 16 lists the duplicates of the events detected by the code questionnaire technique [4, 6]. The personalities listed in one column are duplicates or analogs. No events listed in the first column are duplicates. Their originals (preimages) are probably the events in 13th century Italy. Table 15 describes the parallelism detected by the author between the events in medieval and ancient Greece. They are overlapped by an upward rigid shift of the ancient events by some 1800 years. This table is also totally consistent with the decomposition of the GCC into the sum of four identical chronicle strata C₁, C₂, C₃, C₄. See also [1-4, 6-13, 22].

The tables list the dates of rule, and in parentheses we indicate the length of rule, for example, Arcadius 395-408 (13). In addition to numerical data, the tables reproduce some fragments of the code questionnaires in order to bring out the parallelism of events. Complete tables of code questionnaires are too extensive to be reproduced in full.

Holy Roman Empire of the German Nation, 10th-13th century A.D.

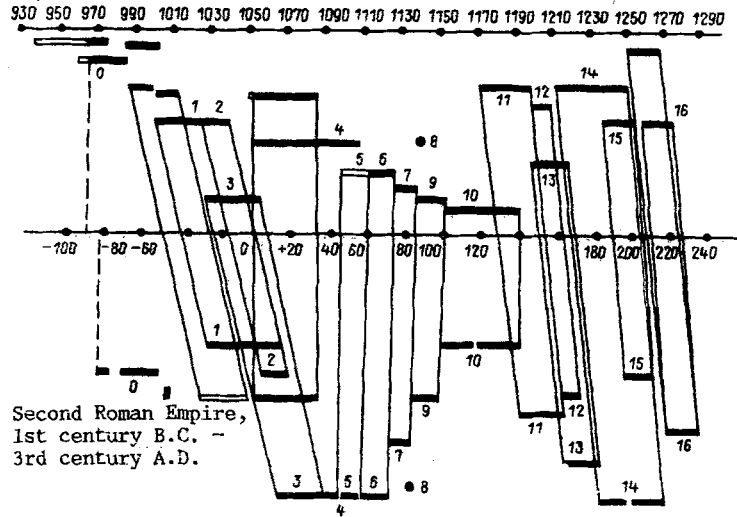


Fig. 9

TABLE 5 (see Fig. 10 below)

Holy Roman Empire of 10th-13th century A.D. 911-1307. The Saxon dynasty begins in 911. Duration of empire 396 years. German rulers identified. Overlap 911 A.D. with 928 B.C. (on the right). For variant rules see [14, 15, 17, 18] (on the left) and [19] or the Bible (references to the Bible denoted by [B])

- 1) Henry I 919-936 (17)
- 2) Lothair 947-950 (3)
- 3) Otto I the Great 936-973 (37)
- 4) Otto II 960-983 (23)
- 5) Otto III the Red 983-996 (13), from accession to the throne in 983 to Roman coronation in 996. For continuation see below
- 6) Otto III from Roman coronation in 996 to 1002, i.e., 996-1002 (6)
- 7) Henry II 1002-1024 - Conrad II 1024-1039 (total 37)
- 8) Henry III 1028-1056 (28)
- 9) Henry IV 1053-1106 (53). Contest with Pope Hildebrand. The excommunication of Henry (Canossa). Acceded to the throne at the age of 6. At the end of his life retired to his secluded castle. Betrayal and coronation of his son Conrad while Henry was still alive. Son rules in father's place (see [17], vol. 5, pp. 233-235)
- 10) Lothair II 1125-1138 (13)
- 11) Conrad III 1138-1152 (14)
- 12) Henry VI 1169-1197 (28). Frederick I attacks Rome in 1167. "Plague" in the German army; they withdraw. Overlap Germany-Assyria (see on the right)
- 13) Frederick II 1196-1250 (54). Famous Roman emperor.
- 14) Conrad IV 1250-1254 (4)
- 15) Charles of Anjou 1254-1285 (31)
- 16) Unrest and events of the period 1285-1307 (22) in Italy (?). End of Empire of 10th-13th century
- 17) Adolf of Nassau 1291-1298 (7)
- 18) Albrecht I 1298-1308 (10)
- 19) Avignon captivity of the Pope (and the papal throne) in France, 1305-Jan. 1376 (70)

The Kingdom of Judah 10th-15th century B.C. [19] existed, according to the Bible, for 395 years. Overlaps with the right column after a rigid shift of approximately 1830 years, i.e., 1838 = 928 + 910. Dates in the right column reckoned from "zero," taking 928 B.C. as the "zero" point. For dates see [19, B]

- 1) Rehoboam 0-17 (17)
- 2) Abijah 17-20 (3)
- 3) Asa 20-55 (35) or 20-61 (41)
- 4) Jehoshaphat 55-79 (24), 61-86 (25)
- 5) Jehoram of Judah (8) by [B] or (6) by [19] + Ahaziah of Judah (1), total (9), (7), i.e., 86-94 by [B]
- 6) Athaliah 96-101 (6). See dates in 2 Chronicles, 1-2 Kings
- 7) Jehoash of Judah 92-130 (38) by [19] or (40) by [B]
- 8) Amaziah 130-159 (29)
- 9) Uzziah from 159 by [19] to 211, ruling (52) by [B] or (43) by [19]; 211-159-52 by [B]. Contest with Chief Priest Azariah. Excommunication of Uzziah. Came to the throne at the age of 16. At the end of his life was a "leper" and lived in a "separate house." His son ruled de facto in his place (2 Chronicles, 26:21-23)
- 10) Jotham 211-227 (16) [B], (7) [19]
- 11) Ahaz 227-243 (16) [B], (20) [19]
- 12) Hezekiah 256-285 (29). The Assyrian king Sennacherib attacks Jerusalem in the reign of Hezekiah, "Plague" in Sennacherib's army; they withdraw, 2 King 19:35-36. Compare Theodoric (6th cent.)
- 13) Menasseh 285-344 (55) [B] or (45) [19]. Famous king
- 14) Amon 340-342 (2)
- 15) Josiah 342-373 (31)
- 16) Jehoahaz (less than 1) + Jehoiakim (11) + Jehoiachin (less than 1) + Zedekiah (11) 373-397 (22) or (24). End of Kingdom
- 17) Jehoiakim 374-385 (11)
- 18) Zedekiah 386-397 (11)
- 19) Babylonian captivity, Babylon ruled by Persia 397-467. Persia-France (?)

By [19], the Kingdom of Judah begins in 928 B.C. Since in our table the zero year of the Kingdom of Judah is 910 A.D., the shift is approximately 928 + 910 = 1838 years, which is close to the third fundamental shift of 1778 (or 1800) years on the GCC [1, 2, 6].

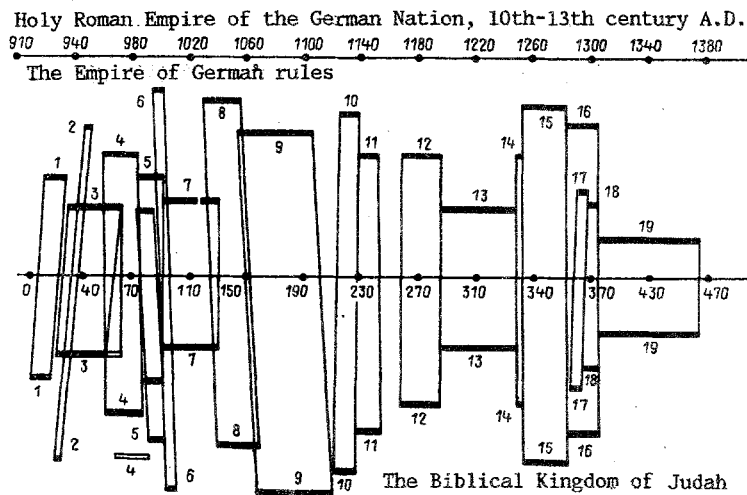


Fig. 10

TABLE 6 (see Fig. 11 below)

Roman coronations of Holy Roman emperors of the German Nation 10th-13th century A.D. Rigid shift of 1840 years. Variant rules see in [14, 15, 17]

- 1) Hugh of Arles 926-947 (21), king of Italy, the beginning of the Empire
- 2) Lothair 947-950 (3), king
- 3) Otto I the Great 936 German coron.-960 beginning of Otto II; or 936 German coron.-962 Roman coron. Thus (24) or (26). Pope John XII - Octavian [17]. See Octavian Augustus.
- 4) 962 Roman coron.-973 German coron. (11); 973-death of Otto I and German coron. of Otto II
- 5) 973 German coron.-996 Roman coron. (23)
- 6) 996 Roman coron.-1014 Roman coron. (18). Complete table of all variants on the right is given in [6].
- 7) 1014 Roman coron.-1027 Roman coron. (13)
- 8) 1014 Roman coron.-1046 Roman coron. (32)
- 9) 1046 Roman coron.-1084 Roman coron. (38)
- 10) 1084 Roman coron.-1125 death of Henry V, end of Franconian dynasty and beginning of Saxon dynasty
- 11) 1125-1134 Roman coron. (9)
- 12) 1134 Roman coron.-1155 Roman coron. (21)
- 13) Pope Alexander III from accession to invasion of Frederick I in 1167, i.e., 1159-1167 (8). In 1143-1155, Italian wars of Germany. In 1154, capture of Rome by Frederick I

Kingdom of Israel (reference to the Bible denoted by [B]) starts at 922 B.C. [19]. For simplicity, years reckoned from zero. Here 922 B.C. = 928 A.D.

- 1) Jeroboam 0-22 (22). The beginning of the Kingdom
- 2) Nadab 22-24 (2) (or Navoth)
- 3) Baasha 24-48 (24). See Table 5; Asa of Judah -duplicate of Otto I. Close names Asa-Baasha (Jesus?) Compare Hildebrand in 11th century A.D. Under Octavian, Jesus born in 1st century A.D.
- 4) Omri 51-63 (12). Dates of all rules according to 2 Chronicles and 3-4 Kings
- 5) Ahab 63-85 (22). "Double counting" in [B] reveals lacunae, see [5].
- 6) Ahaziah (2) + Jehoram of Isarel (12), i.e., 85-99 (14). This is the 1st variant of Jehoram in [B].
- 7) Jehoram of Israel 94-106 (12). 2nd variant of Jehoram in [B].
- 8) Jehu (28) + lacuna (2) (unrest) 99-127-129 (total 30 years)
- 9) Jehoahaz (17) + Jehoash (16), i.e., 127-144-160 (total 33 years)
- 10) Jeroboam II 160-201 (41). Implied superpositions: Assyria-Germany, Persia-France, Babylon-Rome or Avignon, Hittites-Goths
- 11) Menahem 203-213 (10)
- 12) Pekah 215-235 (20)
- 13) Hoshea 235-243 (8). War with Assyria, invasion of Shalmaneser. Superposition of Assyrian on Germany (left). End of the Kingdom of Israel. Superposition of the "pharaohs" from [B] on TRK and TRNK (Franks, Goths)

According to [19], the Kingdom of Israel begins in 922 B.C. Since the zero year of the Kingdom of Israel in the table corresponds to 920 A.D., the shift is approximately $920 + 922 = 1842$ years, which is close to the third fundamental shift of 1778 (or 1800) years on the author's GCC [1, 2, 6].

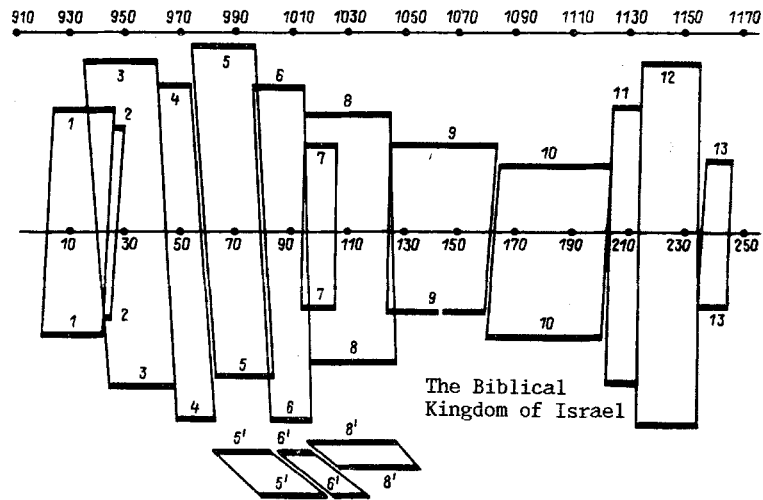


Fig. 11

TABLE 7

1st period of the Roman pontificate 141-314 A.D.
Variant rules see [14,21] (also the right column)

- 1) St. Pius 141-157 (16)
- 2) St. Anicetus 157-168 (11)
- 3) St. Soter (= the savior) 168-177 (9)
- 4) St. Eleutherius 177-192 (15)
- 5) St. Victor 192-201 (9)
- 6) Zephyrinus 201-219 (18)
- 7) Calixtus 219-224 (5)
- 8) Urban I 224-231 (7)
- 9) Pontianus 231-236 (5)
- 10) Fabian 236-251 (15)
- 11) Unrest 251-259 (8)
- 12) Dionysius 259-271 (12)
- 13) Felix I 275-284 (9) or Eutychianus?
- 14) Eutychianus 271-275 (4), Felix I?
- 15) Caius 283-296 (13)
- 16) Marcellinus 296-304 (8)
- 17) Marcellus 304-309 (5)
- 18) Eusebius 309-312 (3)
- 19) Melchiades 311-314 (3)

2nd period of the Roman pontificate 314-532 A.D.
Consistent with superposition of Second and Third Roman Empires

- 1) Sylvester 314-336 (22)
- 2) Julius I 336-353 (17)
- 3) Liberius (= the liberator) 352-367 (15)
- 4) Damasus 385-398 (13)
- 5) Siricius 385-398 (13)
- 6) Anastasius, Innocent 398-417 (19)
- 7) Boniface 418-423 (5)
- 8) Celestine 423-432 (9)
- 9) Sixtus III 432-440 (8)
- 10) St. Leo (Leo I) 440-461 (21)
- 11) Unrest and Hilarius 461-467 (6)
- 12) Simplicius 467-483 (16)
- 13) Felix II 483-492 (9)
- 14) Gelasius 492-496 (4)
- 15) Symmachus 498-514 (16)
- 16) Hormisdas 514-523 (9)
- 17) John I 523-526 (3)
- 18) Felix III 526-530 (4)
- 19) Boniface II 530-532

TABLE 8

Royal Rome, First Empire 753-500 B.C.
See T. Livius. A shift of 1050 years
(approximately)

- 1) Romulus Quirinus (37)
- 2) Numa Pompilius (43)
- 3) Tulla Hostilus (32)
- 4) Ancus Martius (24)
- 5) Tarquinius Priscus (38)
- 6) Servius Tullius (44)
- 7) Tarquinius Superbus (25)
- 8) The fall of Royal Rome, the expulsion of the kings, the war with the Tarquinians around 500 B.C. See below
- 9) The clan of Tarquinians, phonetically TRKVN (without vowels), enemies of Rome. Their duplicates - Trojans (see Troy).
- 10) Valerius - Roman military leader opposing the Tarquinians. Name without vowels VLZR. Valerius accused of treason
- 11) The message of L. Tarquinius Superbus to the Roman Senate. The plot in Rome and its exposure. Upward shift of 1050 years places these events at 543-544 A.D.
- 12) The first battle for Rome in 545. The Tarquinians lose.
- 13) The second march on Rome in 546. Roman leader Lars Porsena. The Tarquinians are crushed.

A strand of the Third Roman Empire 3rd-4th cent. A.D.
Time intervals identified in [6]. Some representatives from each interval are identified

- 1) 300-337 (37), Constantine I
- 2) 337-380 (43), Basil the Great
- 3) 380-423 (43), Honorius
- 4) 423-444 (21), Aecius
- 5) 444-476 (42), Ricimer
- 6) 476-526 (50), Odoacer, Theodoric
- 7) 526-552 (26), the Gothic dynasty
- 8) The fall of Third Roman Empire in the West, expulsion of the Goths. The war with the Goths in 6th cent. B.C. See below
- 9) The clan of Goths, enemies of Rome in the war of 6th century. The allies of the Goths are Franks, phonetically TRK or TRNK (compare "F" and "T").
- 10) Belisarius - Roman military leader opposing the Goths. Name without vowels VLZR (on the left, son of Volusius). Belisarius accused of treason
- 11) The message of Totila the Goth to the Roman Senate. The plot in Rome and its exposure. These events occur in 543 A.D. The dates on the left are shifted by the formula X+300, where X is the year when Rome was founded.
- 12) The first battle for Rome in 545-547. The Goths lose.
- 13) The second march on Rome in 548-549. Roman leader Narses. The Goths are crushed.

TABLE 9 (see Fig. 12 below)

Second Roman Empire from 82 B.C. to 3rd cent. A.D. The first 8 items in this column are approximate, without affecting the closeness of the streaks. Periods of unrest also indicated

- 1) Lucius Sulla 82-78 (5)
- 2) Unrest 78-77 (1)
- 3) Sertorius 78-72 (6)
- 4) Unrest 72-71 (2)
- 5) Pompey the Great 70-49 (21)
- 6) Pompey and Caesar jointly 60-49 (11)
- 7) Unrest 49-45 (4)
- 8) Julius Caesar, the winner of the 1st Triumvirate 45-44 (1)
- 9) Triumvirs and Octavian Augustus 44-27 (17)
- 10) Octavian Augustus 27 B.C.-14 A.D. (41) or (37) from 23 B.C.
- 11) The birth of Jesus in the 27th year of Octavian Augustus (27)
- 12) Tiberius 14-37 (23)
- 13) Tiberius and Germanicus jointly 6-19 (13)
- 14) Caligula 37-41 (4)
- 15) Unrest of 41 (1)
- 16) Claudius 41-54 (13)
- 17) Claudius and Pallans jointly 41-54 (13)
- 18) Nero 54-68 (14)
- 19) Nero-Burrus-Seneca jointly 54-62 (8)
- 20) Galba 68-69 (1)
- 21) Unrest of 69 (1)
- 22) Two Titus Vespasians 69-81 (12). Identical names
- 23) Domitian 81-96 (15)
- 24) Nerva 96-98 (2)
- 25) Joint rule of Nerva 96-98 (2)
- 26) Trajan 98-117 (19)
- 27) Hadrian 117-138 (21)
- 28) Titus Antonius Pius 138-161 (23)
- 29) Marcus Aurelius 161-180 (19)

- 30) Lucius Commodus 176-192 (16)
- 31) Pertinax 193 (1)
- 32) Didius Julianus 193 (1)
- 33) Claudius Albinus 193 (1)
- 34) Pescennius Niger 193-194 (1)
- 35) Septimius Severus 193-211 (18)
- 36) Caracalla 193-217 (24). The famous reforms of the Second Empire
- 37) End of the Second Roman Empire. The crisis of mid-3rd cent. War with the Goths. A shift of 333 years (approximately)

A strand of the Third Roman Empire 3rd-6th cent. A.D. Variant rules see [14, 16, 5, 19]. Dates sometimes reckoned from the death of the co-ruler. Analysis of complete list see [6]

- 1) Lucius Aurelian 270-275 (5)
- 2) Unrest 275-276 (1)
- 3) Probus 276-282 (6)
- 4) Unrest 282-284 (2)
- 5) Diocletian the Great 284-305 (21)
- 6) Diocletian and Constantius Chlorus jointly 293-305 (13)
- 7) Unrest 305-309 (4)
- 8) Constantius Chlorus, the winner of the 1st Tetrarchy 305-306 (1), after No. 5
- 9) Tetrarchs and Constantine Augustus 306-324 (18)
- 10) Constantine Augustus 306-337 (31). Beginning of No. 12 from the death of Constantine
- 11) The birth of Basil the Great in the 27th year of Constantine Augustus (27)
- 12) Constantius II 337-361 (24)
- 13) Constantius II and Constans jointly 337-350 (13) from the end of No. 10
- 14) Julian 361-363 (2) from No. 12
- 15) Unrest of 363 (1)
- 16) Valentinian I 364-375 (11)
- 17) Valentinian and Valens (Pallans?) jointly 367-375 (11)
- 18) Valens 364-378 (14)
- 19) Valens-Valentinian-Gratian jointly 364-375 (11)
- 20) Jovianus 363-364 (1), transposed
- 21) Unrest of 378 (1)
- 22) Gratianus and Valentinian II after Valens and unrest 379-392 (13)
- 23) Theodosius I 379-395 (16)
- 24) Eugenius 392-394 (2)
- 25) Joint rule of Eugenius 392-394 (2)
- 26) Arcadius 395-408 (13)
- 27) Honorius 395-423 (28)
- 28) Aetius 423-444 or 423-438 (14) or (21)
- 29) Valentinian III 437-455 or 444-455 (11) or (18)
- 30) Ricimer 456-472 (16)
- 31) Olybrius 472 (1)
- 32) Glycerius 473, 474 (1)
- 33) Julius Nepos 474 (1)
- 34) Romulus Augustulus 475-476 (1)
- 35) Odoacer 476-493 (17)
- 36) Theodoric 493-526 or 497-526 (33) or (29). Famous reforms
- 37) End of Third Roman Empire in the West. The famous war with the Goths in mid-6th century.

This parallelism is secondary, being a consequence of the main superpositions listed above. Both streaks include additional numerical data which are not taken into consideration when calculating the closeness coefficient of the streaks.

TABLE 10

The Biblical Kingdom of Judah with the capital in Jerusalem, 10th-7th cent. B.C.	Eastern Roman Empire 306-700 B.C., with the capital in New Rome
1) Rehoboam (17)	1) Licinius 308-324 (16)
2) Abijah (3), "the Father of God" - trans.	2) Arius 330-333 (3) (5) (8) (variants)
3) Asa (Jesus?) (46) (41)	3) Basil the Great? 333-378 (45)
4) Jehoshaphat (25)	4) Theodosius I 379-395 (16)
5) Jehoram of Judah (8). Separation of Edom. Then a gap of 76 years (see below)	5) Arcadius 395-408 (13). Separation of the Western Empire from the Eastern Empire
6) Uzziah (52). Involved in a Temple dispute, cursed and struck with leprosy	6) Theodosius II 408-405 + Marcian 450-457 (49), dispute in the Council of Ephesus
7) Interregnum (2). A lacuna in 2 Chronicles	7) The invasion of Attila and anarchy 451-453 (2)
8) Jotham (16)	8) Leo I 457-474 (17)
9) Ahaz (16). Rezin king of Syria and Pekah king of Israel attack Jerusalem and Ahaz turns to Tiglath-pileser for help (a duplicate of Theodoric on the right?)	9) Zeno 474-491 (17). The Germanic leader Odoacer attacks Rome. The Western ruler is Ricimer (Rezin?) 456-472. Zeno turns to Theodoric the Goth for help
10) Hezekiah (29)	10) Anastasius 491-518 (27)
11) Manasseh (55) (50). A famous king, accused of a massacre in Jerusalem (suppressing a rebellion?) Superposition of the capital on New Rome	11) Two Justins: Justin I 518-565 (a total of 47). Suppression of the Nika revolt in New Rome, a massacre
12) A 76-yr insert: 4 kings + Amon (2 yrs) = 5 kings, a total of 78 years, i.e., 78 = 76 + 2	12) 5 emperors: Justin II+ Tiberius II + Mauricius + Phocas + Heraclius, total 565-641 (76)
13) Josiah (31), invasion of the Pharaoh	13) Constant II 642-668 (26), Arabs
14) Jehoahaz (1)	14) Constantine III 641-642 (1)
15) Jehoiakim (11)	15) Constantine IV 668-685 (17)
16) Jehoiachin (1)	16) Heraclion 641-642 (1)
17) Zedekiah (11). Nebuchadnezzar king of Babylon "takes the people into captivity"	17) Justinian II, first reign 685-695 (10). Imperial war, Arabs
18) The end of the Kingdom of Judah, Babylonian captivity	18) The famous crisis of the end of the 7th century A.D. The breakup of the Eastern Empire

This is a secondary parallelism which follows from the parallelisms listed above and in [6]. The shift of some 1300 years is the sum of two shifts 1000 + 300.

TABLE 11

The Biblical Kingdom of Israel 10th-8th cent. B.C. A shift of 1300 years	A strand of the Third Roman Empire 4th-5th cent. A.D. On the left sum of shifts 1000 + 300
1) Jeroboam I, Jeroboam's "heresy," split and war with Rehoboam (22). On the right "heresy" - Arianism?	1) Constantine I after deposing Maxentius 313-327 (24). Rift and war with co-ruler Licinius
2) Nadab (2)	2) Constantine II 337-340 (3) after No. 1
3) Baasha (24). On the right Basil the Great (Jesus-Asa-Baasha?)	3) Constantius II after the death of Constantine II 340-361 (21), then No. 4
4) Elah (Elius?) (2)	4) Julian (Elius?) 361-363 (2)
5) Zimri (1)	5) Jovain 363 (1)
6) Omri (12)	6) Valentinian I 364-375 (11)
7) Ahab (the Wicked) (22), contest with the great prophet Elijah. Mortally wounded in battle	7) Valens (the Wicked) 364-378 (14), contest with the prophet Basil the Great. Killed in battle
8) Ahaziah (2). Rules in Samaria (superimposed) on Rome on the right)	8) Gracianus and Valens and unrest 379-383 (4). Then No. 9 after No. 7
9) Jehoram of Israel (12)	9) Valentinian II 379-392 (13)
10) Jehu and the prophet Elisha (revolution to seize power) (28)	10) Alaric and John the Eloquent 378-403 (25?) (32?)
11) Jehoahaz (17)	11) Theodosius I 379-395 (16)
12) Jehoash of Israel (16)	12) Arcadius 395-408 (13)
13) Jeroboam II (41)	13) Honorius 395-423 (28)
14) Zechariah (6 months) (1)	14) Constantium III 421 (7 months)
15) Shallum (1 month) (1)	15) John 423 (2 months)
16) Interregnum (24)	16) Interregnum, regency 423-444 (21)
17) After the interregnum - Menahem, invasion of king Pul or Tul (10). Superposition of the names Tul (TL) and Attila	17) Valentinian III after regency 444-455 (11), invasion of Attila (TTL)
18) Pekahiah (2)	18) Petronius Maximum 455-456 (1)
19) Pekah (20), invasion of Tiglathpileser (the deporter)	20) Ricimer 456-472 (16), invasion of Genseric, deportation of nations
21) Hoshea (until captivity) (1), invasion of Shalmaneser and capture of Hoshea	21) Romulus Augustulus 475-476 (1). Invasion of Odoacer and capture of Romulus
22) The end of the independent existence of the Kingdom of Israel. Hoshea is the last independent king.	22) The end of the independent existence of the Third Western Roman Empire as a "pure Roman" state.

TABLE 12 (see Fig. 13 below)

First Byzantine Empire 527-829 A.D. Duration of 302 years [14, 20]	Second Byzantine Empire 829-1204 A.D. Duration of 375 years [14, 20]
1) Justinian I Basileus and Theodora 527-565 (38). This is the beginning of the Eastern Roman Empire proper. A rigid shift of approximately 340 years	1) Theophilus 829-842 + Michael III and Theodora 842-867 (38). Then the Macedonian dynasty begins. Compare with biography of Justinian I
2) Justin II 565-578 + Tiberius II 578-582 (17)	2) Basil I (Basileus I) 867-886 (19). Compare Justinian I Basileus
3) Mauritius 582-602 (20)	3) Leo VI 886-912 (26)
4) Phocas 602-610 (8)	4) Alexander 912-913 (1)
5) Heraclius 610-641 (31). Then unrest in No. 6 on both left and right	5) Constantine VII 910-912-959 (47) or (49)
6) Constantine III 641, Heraclion = Heraclius II 641 (total around 1). Beginning of unrest	6) Romanus II 959-963, Nicephorus II Phocas 963-969, John I Tzimisce 963-975 or 976 (total 16)
7) Constans II 642-668-Constantine IV 668-685 - Justinian II 685-695 (53)	7) Constantine X or VIII 975-1028 (53), different versions
8) Unrest: Leontius II 695-698 or 694-697, Tiberius III Apsimar 697-704 or 698-705, Justinian II (second time) 705-711, Philippicus Vardanus 711-713, Anastasius II 713-715 or 716, Theodosius III from 715 or 761 to 717. Total (22)	8) Unrest: Constantine VIII 1025-1028, Romanus III 1028-1034, Michael IV 1034-1041, Michael V 1041-1042, Constantine IX Monomachus 1042-1054, Theodora 1054-1056, Michael VI 1056-1057. Total (20). To overlap with the unrest on the left: a shift of 340 years.
9) The war of Justinian II (see above) - a partial duplicate of the Gothic-Trojan-Tarquinian war, abbreviated as GTR-war	9) In 1047 the revolt of Tornika (Nika TR?) Compare with the Nika revolt in the time of Justinian I. The Nika revolt - a duplicate of the GTR-war by [6].
10) Leo III the Isaurian 717-741 (24)	10) John II Comnenus 1118-1143 (25)
11) Constantine V Kopronymus 741-775 (34)	11) Alexius I Comnenus 1081-1118 (37), transposed with No. 10 (John)
12) Leo IV 775-780 + Constantine VI 780-797 + Irene 797-802 + Nicephorus 802-811 (36)	12) Manuel I Comnenus 1143-1180 (37)
13) Unrest: Stauracius 811, Michael I Rhangabe 811-813, Leo V 813-820 or 821, Michael II 820 or 821-829. Total (19). In both columns numerous periods of unrest, thus complicating the overall picture. Both Empires overlapped by a shift of 340 years.	13) Unrest: Alexius II Comnenus 1180-1183, Andronicus I 1183-1185, Isaac II Angelus 1185-1195, Alexius III 1195-1203, Alexius IV 1203-1204, Isaac II Angelus (second time) 1203-1204, Alexius V 1204. Total (24). Fall of Constantinople in 1204.

TABLE 13 (see Fig. 13 below)

Third Byzantine Empire 1204-1453 A.D. Duration 249 years, numerous periods of unrest. Both empires overlap after a shift of 340 years	Second Byzantine Empire, the strand from Basil I to John VIII, i.e., 867-1143 A.D. Duration 276 years. Variant rules see [14, 20]
1) The beginning of the Nicaean Empire in 1204. See Nika in the GTR-war of 6th century A.D. Theodore I Lascaris 1204-1222 (18)	1) Basil I (Basileus I). Nika revolt in the time of Justinian I in the 1st Byzantine Empire. 867-886 (19). Theodora was the wife of Justinian I
2) John III Dukas Vatatzes 1222-1254 or 1256- (32). The GTR-war	2) Leo VI the Wise 886-912 (26)
3) Theodore II Lascaris 1254 or 1256-1258 or 1259 (3)	3) Alexander 912-913 (1)
4) Michael VIII 1259 or 1260-1282 or 1283 (25)	4) Romanus I 919-945 (26)
5) Andronicus II Paleologus 1282 or 1283-1320 or 1328 (46)	5) Constantine VII 910 or 912-959 (47)
6) Andronicus III Paleologus 1320-1341 (21)	6) Romanus II + Nicephorus II Phocas + John I Tzimisce 959-976, 975 (16) or (17)
7) 2nd variant: Andronicus III Paleologus 1328-1341 (13)	7) Nicephorus II Phocas + John I Tzimisce 963-976 (13), 2nd variant
8) John V Paleologus 1341-1391 or 1376 (50)	8) Basil II Bulgaroktonus 975 or 976-1025 (50)
9) Unrest 1376-1391. Andronicus IV 1376-1379, John V (second time) 1379-1391, John VII 1390 (15)	9) Unrest 1025-1057. For the list of emperors in this period see Table 12
10) Manuel II 1391-1424, 1425 (34)	10) Alexius I 1081-1118 (37)
11) John VIII by [20] or VI by [14] 1424 or 1425-1448 (24). The fall of Constantinople in 1453. The end of the Byzantine Empire	11) John II 1118-1143 (25) (Nos. 10 and 11 both Comneni). Followed by Manuel I and the unrest of 1180-1204 (nor included in the parallelism). The fall of Constantinople in 1204

TABLE 14a

Traditional chronology. Chronicle E on the GCC in [2], years B.C. Conventional "textbook" representation	Biblical chronology. Chronicle B on the GCC, years B.C. A shift of 1800 years upward in time
<p>1) K 1460-1236. The Trojan Kingdom of the Seven Kings, Trojans, Greeks.</p> <p>2) T 1236-1226. The Trojan War (Greece?). The expulsion of the Trojans, the fall of Troy.</p> <p>3) H 1226-850. The kings of Ancient Greece</p>	<p>The dates of the corresponding events are shifted upward because of the parallelism discovered by the author. In addition to the shift, there is an identification with the left column which results in overall shortening.</p>
<p>4) T 850-830. 2nd version of the dating of the Trojan War by T. Livius. The apple of contention of Venus (Eve?)</p> <p>5) T 760-753. The founding of Rome, Romulus and Remus, the kidnapping of the Sabines</p> <p>6) K/P 753-522. Royal Rome of the Seven Kings according to T. Livius. Great Greek colonization 8th-6th century</p> <p>7) T 522-509. The war with Tarquinians. Deposing of the kings, the beginning of republican Rome.</p>	<p>4) T 850-830. Genesis 1-3. Adam and Eve, the apple of contention, the expulsion from Paradise</p> <p>5) T 760-753. Genesis 4:1-16. Cain and Abel, the murder of Cain</p> <p>6) K 753-522. Genesis 4-5:31. Enoch, Irad, Mehujael, Methuselah, Lamech, Seth, Enosh, Kenand, Mahalelel</p> <p>7) T 522-509. Genesis 5:32-8. The legend of Noah and the Flood, the destruction of mankind, the ark, the covenant</p>
<p>8) H/C 509-82. Ancient republican Rome. Greco-Persian wars. Cyrus, Darius, Xerxes. Peloponnesian Wars. Macedonians, Philip II. The fall of Byzantium. The Empire of Alexander the Great. A famous period in the history of classical Greece. Roman wars with Samnites. Punic wars. Hannibal. The end of classical Greece. The beginning of Hellenism.</p>	<p>8) H 509-82. Genesis 9, 10:1-32. The descendants of Noah, peopling of the earth. Noah's sons Shem, Ham, and Japheth. Japheth's sons. Comment to No.7: There is a parallelism between Noah and Moses, duplication of the same term "the ark of covenant" (Moses) and "the ark" (Noah), duplication of covenants. A legend that Noah founded a city near Rome (see [17], Vol. 3, p. 437).</p>
<p>9) T 82-83. The beginning of Imperial Rome. Sulla, Pompey, Caesar, Octavian Augustus, civil wars of 1st century.</p>	<p>9) T 82-23. Genesis 11:1-9. Babylonian confusion, the scattering of the people, chaos.</p>

Chronicle C₄ on GCC, years A.D. A shift of 1778 (approximately 1800) years downward in time

- 1) K 306-535. Third Roman Empire 4th-6th century, both East and West.
- 2) T 535-552. The Gothic war in Italy, the expulsion of the Goths, the fall of Naples and Rome
- 3) H 552-901. Medieval Papal Rome, Greece

4) T 901-924. The war in Italy. Alberic I and Theodora I. The legend on "the woman of contention."

5) T 931-954. The War in Italy. Alberic II and Theodora II

6) P 962-1250. Holy Roman Empire of the German Nation 10th-13th century. Crusades

7) T 1250-1268. The war in Italy. Manfred, Conrad, the fall of medieval Troy.

8) C 1300-1550. The Habsburg Empire. Medieval Greece and the battles of 1316 (the original of the battle of Marathon). Duke Walter II. The wars between the Franks and the Turks. Mohammed II, Mohammedans. The fall of Byzantium in 1453. The Ottoman Empire. The end of independent existence of medieval Greece

The end of chronicle (line) C₄. We described the skeletal events which moved down as a result of the shift of 1778-1800 years. The reason for this shift - the use of abbreviations to record dates.

Chronicle C₃ in GCC, years A.D. A shift of 1053 years downward in time

The reason for the 1000-year shift is the following: some dates were originally recorded as X.1 century, which stands for the 1st century of Christ (X = Christus). Later the meaning of this X was forgotten, and the formula "X.1 century" was formally decoded as "X1 century," i.e., 11th century. Similarly the year I.100 stood for year 100 of Jesus (I = abbreviation of Jesus), but was eventually decoded as the year 1100, since the letter I also stood for "one thousand." As a result, these dates were shifted by 1000 years.

6) K 300-535. Roman Empire of 4th-6th century. In 325, the founding of New Rome. The Eastern campaigns of Rome

7) T 535-552. Gothic war in Italy. The fall of Naples and Rome. Justinian, Belisarius, Narses. Goths, Franks -TRN.

8) H 552-901. Medieval Papal Rome. The wars of the Romans with Lombards 705, 711 and later until 765 and 769. The war in the south of Italy. The wars with Saracens. Frankish wars in Italy. Comment to No. 13 in chronicles G and E on the left: Charlemagne - Joshua the son of Nun, the defeat of Roland - the defeat of the army near the city of Ai; both Charlemagne and Joshua stopped the sun during their battle (unique episodes), the traitor Ganelon - the "traitor" Achan.

9) T 931-954. The wars in Italy. Alberic II and Theodora II. Restoration of many ancient customs. The beginning of the Holy Roman Empire of the German Nation

TABLE 14b

<p>Traditional chronology. Chronicle E on GCC, years A.D.</p> <p>10) P/K 23 B.C.-235 A.D. Second Roman Empire 1st-3rd century. The beginning of the "new era," religious re- forms, Jesus Christ</p>	<p>Biblical chronology. Chronology B on GCC, years A.D. after a shift of 1800 years</p> <p>10) K 23 B.C.-217 A.D. Genesis 11:10-32. Ar- pachshad, Shelah, Eber, Peleg, Reu, Serug, Nahor, Terah, Haran, Abram. Haran - Arius?</p>	<p>Chronicle C₃ on GCC, years A.D. after a shift of 1053 years</p> <p>10) P 965-1250. Holy Roman Empire of the German Nation. The beginning of a "new era" under Pope Hil- debrand. Rift</p>	<p>Chronicle C₂ on GCC, years A.D. after a shift of 333 years</p> <p>10) K 306-535. Roman Empire 4th-6th century. In the 6th century, Basil the Great and his religious reforms. A rift in the church, Arius</p>	<p>Chronicle C₁ on GCC, years A.D. Distorted original, no shift</p> <p>This chronicle has not yet been moved down, but it already con- tains a small number of duplicates</p>	<p>Chronicle C₀ on GCC, years A.D. Original, no shift.</p> <p>This column con- tains a part of the modern "text- book." This part served as the original for the downward shifted chronicles C₂, C₃, C₄. There are virtually no his- torical data be- fore 10th century</p>
<p>11) T 235-251. Julia Maesa, anarchy, Gothic war. I 270-300. War</p> <p>12) π/K/P/C 300-535. Roman Empire of 4th- 6th century. Division into two kingdoms: East and West</p> <p>13) T 535-552. The Gothic war in Italy. The end of the Roman Empire. The wars of Charlemagne - the wars of Joshua.</p> <p>14) π/H/P 566-901. Medieval Papal Rome. Carolingians, the empire of Charlemagne</p>	<p>11) T 270-300. Abraham and Sarah. The con- frontation with Pharaoh - TRN. Genesis 12</p> <p>12) K 306-535. Genesis 13-38. Abraham and Haran. Division into two kingdoms. Isaac, Esau, Jacob, Joseph</p> <p>13) T 535-552. Genesis 39-50. Exodus (Moses). Leviticus. Numbers. Deuteronomy. Joshua. Chanson de Roland.</p> <p>14) π/H/P 556-901. 18. History of the judges.</p>	<p>11) T 1250-1268. The famous war in Italy. The fall of Troy and Naples.</p> <p>12) C 1273-1619. The Habsburg (Roman) Em- pire. Eastern Roman (Byzantine) Empire</p> <p>13) End of chronicle (line) C₃</p>	<p>11) T 535-552. The fa- mous Gothic war in Italy. The fall of Naples, Rome</p> <p>12) π 681-887. Carolin- gians, the empire of Charlemagne. Eastern Roman Empire</p> <p>13) T 901-924. War in Italy. Alberic I, Theodora I.</p>	<p>12) P/K 300-535. Roman Empire of 4th-6th cen- tury. The division of the empire into two: East and West.</p> <p>13) P/K 535-552. Gothic war in Italy. The ex- pulsion of Goths from Italy</p>	<p>12) K 306-526. Few remnants of tra- ditional data about the period of 4th-6th century.</p> <p>13) Insignificant remnants of tra- ditional data about the 6th century.</p>
<p>14) π/H/P 566-901. Medieval Papal Rome. Carolingians, the empire of Charlemagne</p>	<p>14) π/H/P 556-901. 18. History of the judges.</p>	<p>Judges 1- Empire of 10th-13th century</p>	<p>Roman German gians, the empire of Char- lemagne</p>	<p>14) π/H 552-901. Carolin- gians, the empire of Char- lemagne</p>	<p>14) π/H 552-901. Insignifi- cant remnants of data about 6th-9th centuries.</p>

<p>15) T 901-914-924. War in Italy. Alberic I, Theodora I. T 931-954. War, Alberic II, Theodora II</p> <p>16) P/C 962-1250. Holy Roman Empire of the German Nation. Emperors crowned with two crowns: Roman and German (duality)</p>	<p>15) T 901-924. War with the sons of Benjamin. T 931-954. Ruth. 1-2 Samuel. 1 Kings 1-11. 1-2 Chronicles 1-9. Saul, Samuel, David, Solomon</p> <p>16) P 962-1250 1 Kings 12-22, 2 Kings 1-23, 2 Chronicles 10-34. Kingdoms of Israel and Judah (duality)</p>	<p>15) T 1250-1268. The famous war in Italy. The fall of the Hohenstaufens, the fall of Troy, Naples. Manfred, Charles of Anjou, Conrad. Medieval legends about Troy in Italy.</p> <p>16) C 1273-1619. The Habsburg Empire. Renaissance in Europe, revival of ancient motifs. End of chronicle C₂</p>	<p>15) T 901-924. War in Italy. Alberic I, Theodora I</p> <p>16) P 962-1250. Roman German Empire. Rich data. "Doubtful empire."</p>	<p>15) Insignificant remnants of data about the 1st half of 10th century.</p> <p>16) P 962-1250. Roman German Empire. Rich data.</p>
<p>17) T 1250-1268. The famous war in Italy. The fall of the Hohenstaufens. The fall of Troy, Naples. Charles de Anjou, Manfred.</p>	<p>17) T 1250-1268. 2 Kings 24-25, 2 Chronicles 35-36. The war with Pharaoh, Nebuchadnezzar. The fall of the Kingdom of Judah</p>	<p>Possible reason for the shift of 333 years: the date "3rd year of Maximilian" could have been written as MCL.III, i.e., Maximus Caesar</p> <p>Leo. Decoding gives the date 1153 A.D., which differs from the true date of 1496 by 343 years. Recall that Maximilian I 1493-1519.</p>	<p>17) T 1250-1268. War in Italy. The fall of the Hohenstaufens. The fall of Troy, Naples. Rich data</p>	<p>17) T 1250-1268. War in Italy. The fall of the Hohenstaufens. The fall of Troy, Naples. Rich data</p>
<p>18) C 1273-1619. The Habsburg Empire. Avignon captivity of the Pope in France 1305-1378, 70 years. Then return to Rome in Italy</p>	<p>18) C₂ 1273-1400. The books of Ezra, Nehemiah, Esther. Babylonian captivity, under Persian rule, 70 years. Then return to Jerusalem.</p>		<p>18) C 1273-1619. The Habsburg Empire. The chronologist Scaliger, Petavius 16th-17th century. Dionysius Petavius -- the original of Dionysius Exiguus (6th century). The beginning of real history.</p>	<p>18) C 1273-1619. The Habsburg Empire. Council of Trent creates a global chronology and a Biblical canon.</p>

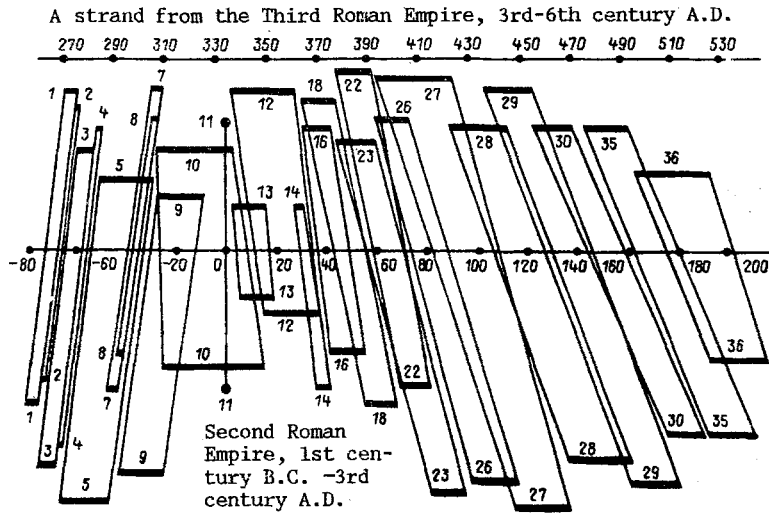


Fig. 12

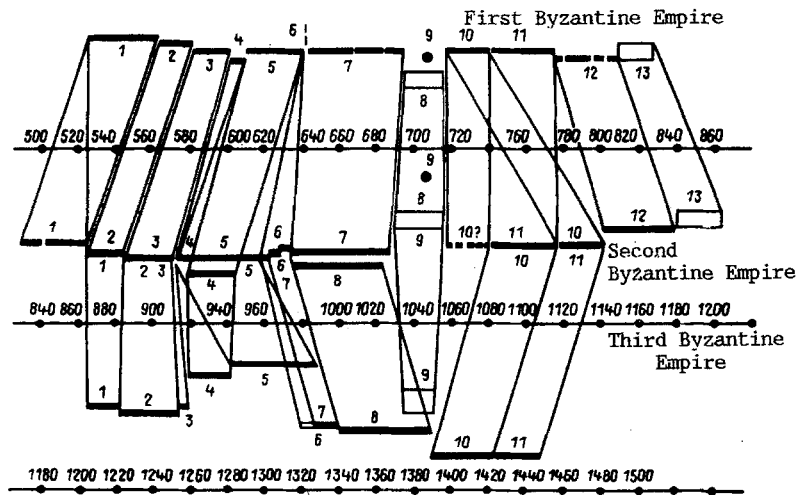


Fig. 13

TABLE 15

Medieval chronology 10th-16th century A.D. Downward shift by 1810 years	Ancient chronology 10th-3rd century B.C. The history of Ancient Greece [14, 20]
1) The Crusades 10th-13th century. Colonization of the Mediterranean	1) The great Greek colonization of 8th-6th century B.C.
2) Holy Roman Empire of the German Nation 911-1305 A.D.	2) Kingdoms of Judah and Israel, 928-531 B.C.
3) Two wars in Italy 10th century A.D.: 901-924 and 931-954. Alberic I, Theodora I, Alberic II, Theodora II	3) Trojan War is dated by T. Livius at 850-830 B.C. This is the 2nd version of its dating
4) The famous war in 13th century Italy: 1250-1268. The fall of the Hohenstaufens, the fall of Troy, Naples. Manfred, Charles of Anjou, Conradin. The accession of the Anjou line. The fall of the pontificate in Rome	4) The war with the Tarquinians in Rome 522-509 B.C. The tyranny of Pisistratus (TRN) 560-527 B.C. The fall of the Pisistratus tyranny 510-514 B.C. The war of Zedekiah (in the Bible) with Pharaoh (TRN). Above TRN - Franks.
5) Avignon captivity 1305-1376, 70 years	5) Babylonian captivity 531-461 B.C., 70 years
6) Wars in medieval Greece 1314-1332, 18 years	6) Greco-Persian wars 492-479 B.C., 13 years
7) War in Greece 1374-1387, 13 years. The eclipse of Thucydides	7) Peloponesian War 431-404 B.C., 27 years
8) The rise and fall of Navarre and the Mistra despotate 1400-1450	8) The rise and fall of Sparta 400-360 B.C.
9) Ottoman Empire 1298-1451. Mohammadans and expansion	9) Macedonian Kingdom 540-359 B.C.
10) The fall of Constantinople and Byzantium in 1453. War with Mohammedans, then fall of Greece	10) From the fall of Byzantium in 364 B.C. and the siege of Byzantium by Macedonians in 340 B.C. (Philip II)
11) Ottoman Empire 15th-16th century.	11) The empire of Alexander the Great 4th-3rd century B.C. Hellenism, the spread of Greek culture through the Mediterranean. End of classical Greece. Items Nos. 12-19 contain addenda
12) Charles of Anjou 1254-1285 (31). In 1268 Charles captures Italy. Without vowels TL-LT	12) Cyrus I 560-530 B.C. (30). In 546 Cyrus captures Lydia. Without vowels LD or TL?
13) Manfred (Kaiser - KZR) 1254-1266. Ruled for 12 years	13) Croesus (KRZ) 560-546 B.C. Ruled for 14 years
14) Charles II of Naples 1285-1289 (4). Bis = the second	14) Cambyses (KM-bis, i.e., KM the second) 530-522 B.C. (8)
15) Frederick II of Sicily 1302-1337 (appr.) (35). In his time, Ferdinand, Margarita (MR - donna), Mathilda	15) Darius I Hystaspes 521-486 B.C. (35). In his time, Artaphernus, Mardonius (MR-donna?), Miltiades (Mathilda?)
16) Duke Walter II de Briehne (HRZG for Herzog = Duke) 1337-1356 (19)	16) Xerxes the Great (XRXS - Herzog?) 486-464 B.C. (22)
17) The restoration of the Parthenon at the end of 14th century A.D.	17) The construction of the Parthenon in 447 B.C.
18) The death of Pleton in 1450	18) The death of Plato in 347 B.C.
19) Mohammed II the Conqueror 1451-1480 A.D. (29)	19) Philip II the Conqueror 359-336 B.C. (23)

TABLE 16

1) Trojan war 13th cent. B.C.	Odysseus (Ulysses) (= Achilles?)	Agamemnon	Achilles	Patroclus
2) Tarquinian War 6th cent. B.C. in Rome	Larcus (- Marcius Coriolanus)	Tarquinius Superbus	Valerius	Junius, son of Marcus Brutus
3) Civil war 1st cent. B.C. in Rome	Sulla and Cicero (NRCC)	Pompey the Great	Julius Caesar	Marcus Brutus
4) Civil war 3rd cent. A.D. in Rome	Aurelian Lucius	Diocletian the Great	Constantius Chlorus	
5) Gothic war 6th cent. A.D. in Rome	Narses (Narcus)	Justinian and Theodora	Belisarius	John II
6) Civil war 901-924 A.D. in Rome	Alberic I? Marotia?	Theophylactus Theodora I	Alberic I	John X
7) Civil war 931-954 A.D. in Rome		Hugh Theodora II	Alberic II	John XI
8) The beginning of the Roman Empire 10th-13th cent. A.D.	Otto I, Otto II, Otto III	Alberic II, Octavian Augustus		
9) War in Italy 13th cent. A.D. Fall of Troy	Charles of Anjou (NRCC)	Innocent IV	Charles of Anjou?	John XXI

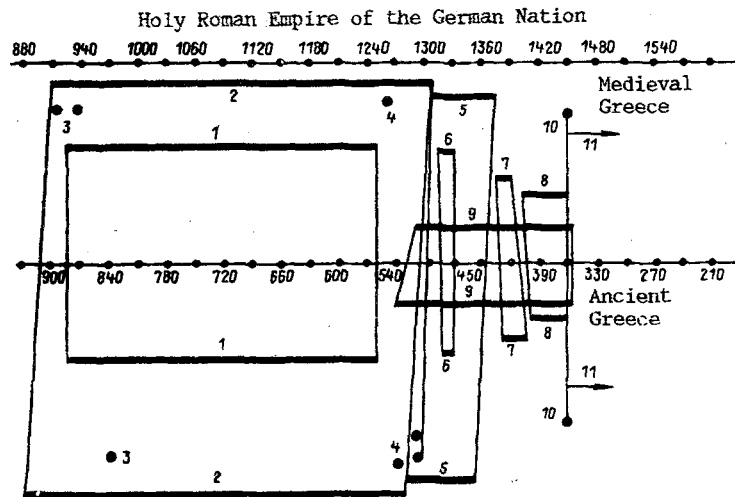


Fig. 14

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SEMIINVARIANTS IN THE THEORY OF CHARACTERIZATION OF
PROBABILITY DISTRIBUTIONS*

Huang Heu Ngy

1. Introduction

In this article we consider semiinvariants, or cumulants — a well-known concept in probability theory. In fact, not all the results are new. They are presented mainly in order to demonstrate the role of semiinvariants.

In what follows, we are dealing with a real random variable (r.v.) ξ whose distribution function (d.f.) is denoted by $F(x) = P(\xi < x)$ and the characteristic function (c.f.) by $f(t)$. We assume that moments of all orders exist for ξ , i.e.,

$$E|\xi|^k < \infty, \quad k=1, 2, \dots$$

We recall that a semiinvariant of order k ($k = 1, 2, 3, \dots$) is defined as

$$\chi_k = \frac{1}{i^k} \left. \frac{d^k \ln f(t)}{dt^k} \right|_{t=0}$$

Therefore with each r.v. ξ is associated a sequence of semiinvariants (s.s.) $(\chi_1, \chi_2, \chi_3, \dots)$.

2. Theorems of Characterization of Probability Distributions in
Terms of Semiinvariants

We first give the characterization theorems for the best known distributions. The proof of the theorems is based on the uniqueness theorem for analytical functions and is therefore omitted.

THEOREM 1. The r.v. ξ is degenerate at the point α , i.e., $P(\xi = \alpha) = 1$, if and only if its s.s. has the form $(\alpha, 0, 0, \dots)$.

THEOREM 2. The r.v. ξ is normal with the parameters (α, σ^2) if and only if its s.s. has the form $(\alpha, \sigma^2, 0, 0, \dots)$.

THEOREM 3. The r.v. ξ is Poisson with the parameter λ , i.e., $P(\xi = k) = \lambda^k e^{-\lambda}/k!$, $k \geq 0$, if and only if its s.s. has the form $(\lambda, \lambda, \lambda, \dots)$.

THEOREM 4. The r.v. ξ is exponential with the parameter $\theta > 0$, i.e., $P(\xi < x) = \max(0, 1 - \exp(-x/\theta))$, $-\infty < x < \infty$, if and only if its s.s. is $(\theta, \theta^2, 2\theta^3, \dots, (k-1)!\theta^k, \dots)$.

THEOREM 5. The r.v. ξ is binomial with the parameter p , i.e., $P(\xi = 0) = p$, $P(\xi = 1) = q$, $p + q = 1$, $p, q > 0$, if and only if its semiinvariants are given by

$$\chi_k = \sum_{r=1}^k s_{kr} q^r, \quad k \geq 1,$$

where

$$\begin{aligned} s_{k1} &= 1, \quad k=1, 2, \dots, \\ s_{kj} &= (-1)^{j-1} \{ (j-1) |s_{k-1, j-1}| + j |s_{k-1, j}| \}, \quad 2 \leq j \leq k-1, \\ s_{kk} &= (-1)^{k-1} (k-1)!. \end{aligned}$$

*Presented by V. V. Kalashnikov.